

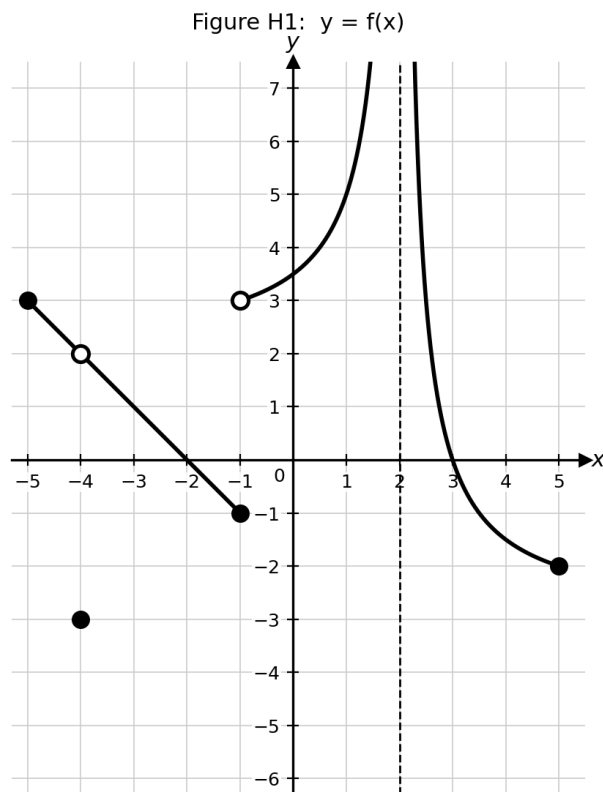
Practice Test H: Limits, Continuity, IVT

Name: _____

Time: 60 minutes No calculator 100 points

Answer exactly the question asked. Show work on every problem in Parts III–V.
 If a limit is infinite, write ∞ or $-\infty$. If a limit does not exist for any other reason, write DNE.
 If a function value does not exist, write *undefined*.

Part I. Graph. Questions 1(a)–1(g) refer to Figure H1.



1. Use the figure to find each value.

(a) $\lim_{x \rightarrow -1^-} f(x)$ _____

(b) $\lim_{x \rightarrow 2^-} f(x)$ _____

(c) $f(2)$ _____

(d) $\lim_{x \rightarrow 2} f(x)$ _____

(e) $\lim_{x \rightarrow -4} f(x)$ _____

(f) $f(-4)$ _____

(g) $\lim_{x \rightarrow -1} f(x)$ _____

(2 pts each, 14 pts)

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Part II. Table.

2. The table gives values of g . Assume the pattern continues as x approaches 2. **(2 each, 6 pts)**

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$g(x)$	1	100	10000	2	10000	100	1

(a) $\lim_{x \rightarrow 2^+} g(x)$ _____ (b) $\lim_{x \rightarrow 2} g(x)$ _____ (c) $g(2)$ _____

Part III. Algebra. Show work.

3. Evaluate $\lim_{x \rightarrow 0} \frac{x^2 - 4}{x^2 - x - 2}$, if it exists. **(5 pts)**

4. Evaluate $\lim_{x \rightarrow 14} \frac{\sqrt{x+2} - 4}{x - 12}$, if it exists. **(5 pts)**

5. Evaluate $\lim_{x \rightarrow -1} \frac{x^3 - 8x - 7}{x^2 + x}$, if it exists. **(6 pts)**

6. Let $R(x) = \frac{x - 5}{x^2 + x - 12}$. Evaluate each limit. **(3 each, 9 pts)**

(a) $\lim_{x \rightarrow 3^-} R(x)$ (b) $\lim_{x \rightarrow 3^+} R(x)$ (c) $\lim_{x \rightarrow 3} R(x)$

7. Let $q(x) = \begin{cases} 2x - 4 & x \leq 2 \\ 3x & x > 2 \end{cases}$ (2 each, 10 pts)
- (a) $\lim_{x \rightarrow 4} q(x)$ (b) $\lim_{x \rightarrow 2^-} q(x)$ (c) $\lim_{x \rightarrow 2^+} q(x)$ (d) $\lim_{x \rightarrow 2} q(x)$ (e) $q(2)$

8. Let $r(x) = \frac{x^3 - 27}{x - 3}$. Find $r(3)$, if it is defined. (3 pts)

Part IV. Continuity. For 9 and 10, check each of the three conditions in the definition of continuity at a point and state your conclusion.

9. Using Figure H1, determine whether f is continuous at $x = -4$. (6 pts)

10. Let $h(x) = \begin{cases} 2x + 5 & x < 2 \\ 3x + 3 & x > 2 \end{cases}$
- Determine whether h is continuous at $x = 2$. (6 pts)

11. Using Figure H1, is f continuous on the closed interval $[-3, -1]$? Explain. (4 pts)

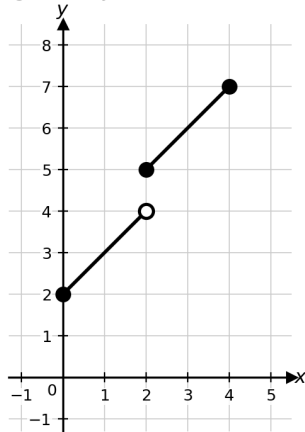
12. Let $u(x) = \frac{x-1}{2x^2-11x+5}$. (3 each, 6 pts)
- (a) Is u continuous on $(5, 7]$? Explain. (b) Is u continuous on $[5, 7]$? Explain.

Part V. Intermediate Value Theorem.

13. The function $h(x) = x^3 - 3x^2 - x + 3$ is continuous on $[-2, 2]$. Find $h(-2)$. (2 pts)
14. Let $f(x) = x^2 - x$ on the interval $[2, 5]$. Does the Intermediate Value Theorem guarantee a number c in $[2, 5]$ with $f(c) = 6$? If it does, find every such c in $[2, 5]$. (6 pts)

15. Figure H2 shows the graph of s on its domain $[0, 4]$. Does the Intermediate Value Theorem guarantee a number c in $(0, 4)$ with $s(c) = 4.5$? If it does, find c . Explain. (4 pts)

Figure H2: $y = s(x)$, domain $[0, 4]$



16. True or False. Circle one. (4 each, 8 pts)
- (a) **T** **F** If f is continuous on $[1, 4]$, $f(1) = 3$, and $f(4) = 3$, then there is no number c in $[1, 4]$ with $f(c) = 11$.
- (b) **T** **F** If g is continuous on $[1, 3]$, $g(1) = 1$, and $g(3) = 9$, then there is at least one number c in $(1, 3)$ with $g(c) = 4$.